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Centrifuge Modeling for Dynamic Geotechnical Studies

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Centrifuge Modeling for Dynamic Geotechnical Studies

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SYNOPSIS The field of dynamic centrifuge modelling has developed rapidly over the past decade, with international interest in both earthquake and blast modelling. The paper discusses the proliferation of facilities and the insight that can be gained from the interpretation of dynamic model test data now available using hand calculations and advanced techniques of digital signal processing.

INTRODUCTION

The period 1980 to 1990 saw a dramatic shift towards the widespread use of the geotechnical centrifuge as a tool for applied research and design. Centrifuges were installed and commissioned worldwide and new research groups have made important contributions to the developing technology. Centrifuges designed specifically for geotechnical application are now available commercially. The proliferation of centrifuges led to a special session at the International Conference in San Francisco (XI International Conference of Soil Mechanics and Foundation Engineering 1985) and to specialist conferences in Paris (1988) and in Boulder, Colorado (1991).

A priority for many of the new facilities has been the study of earthquake effects and dynamic loading. Centrifuge centres, particularly in the UK, Japan and the US have developed shaking tables and fast data aquisition systems for the study of earthquake geotechnical engineering. Experiments in blast modelling and impact have been carried out at rather fewer institutions but with a longer history than the earthquake work, dating back over 50 years.

At the 1981 Missouri Conference Schofield (1981) restated the scaling relations for dynamic modelling at high gravities. In this State-of-the-Art paper the scaling relations for centrifuge work will not be discussed except in the most general terms and the reader is referred to a series of recent papers which describe scaling relations for geotechnical model testing in great detail, including Schofield (1980)(1981), Schofield and Steedman (1988), Iai (1989), Scott (1989) and Schmidt and Holsapple (1980) in the field of explosive modelling.

Two important reference works for centrifuge modellers emerged during the 1980's; these were the State-of-the-Art Volume "Centrifuges in Soil Mechanics" produced for the XI ICSMFE in San Francisco in 1985, Craig et al. (1988), and the Proceedings of the Paris Conference "Centrifuge 88", Corte (1988).

This paper will discuss the developments that have taken place over recent years and the opportunities that have now arisen for the geotechnical community. Ten years on from the Missouri Conference in 1981 the centrifuge has emerged from the research laboratory, accepted as a valuable tool for addressing a wide range of complex and challenging field problems.

EARTHQUAKE AND BLAST

A particular subset of these problems has been in the area of dynamics, including earthquakes and blast loading.

One of the first motivations in the area of earthquake modelling was to model the process of liquefaction and this was achieved at an early stage by a group of researchers including Whitman et al. (1981) and Heidari and James (1982). In recent years attention has shifted towards complex problems of earthquake induced soil-structure interaction, including the behaviour of anchored quay walls and piled structures, Steedman et al. (1990).

A similar shift of emphasis has dominated recent research thrusts in centrifuge modelling of blast problems. Outside the USSR where there has been a long history of centrifuge modelling of blast events, research during the 1970's in the US and the UK was focussed on cratering problems, Schofield (1981).

In recent years centrifuge model tests have been reported on the effects of blast loading on buried structures, using instrumentation to observe structural response.

An increasing sophistication in centrifuge model tests has therefore been observed in both earthquake and blast modelling over the past ten years. The widespread interest in earthquake modelling is best illustrated by the huge investment internationally in dynamic actuators or 'shakers'.

SHAKERS

Since the first steps towards the development of dynamic actuators for use on a centrifuge there has been controversy over both the ideal type of system and the preferred class of input to a model. Clearly these are linked. If a modeller is seeking a random broad band shaking input to a model this will restrict the available choices of actuator.

Historically, design approaches for earthquake loading on geotechnical structures have followed a pseudo-static approach, in which a soil layer is assumed to be rigid, such that vertically propagating elastic shear waves pass instantly through it without amplification or phase change. In physical terms this is broadly equivalent to tilting the layer, an approach adopted for centrifuge model tests of dams by Mikasa et al. (1969).

Fig.1 reproduces Mikasa's diagram of the 200g, 1m radius Osaka City University centrifuge. A simple mechanism using a motor and lead screw arrangement, located on the vertical axis of the centrifuge, moves the main booms relative to each other and tilts the specimen container. As the centrifuge is exactly balanced very little power is required to generate a maximum tilt equivalent to 30% lateral acceleration on 29 kg of soil at 200g (a 0.4 KW motor is quoted).

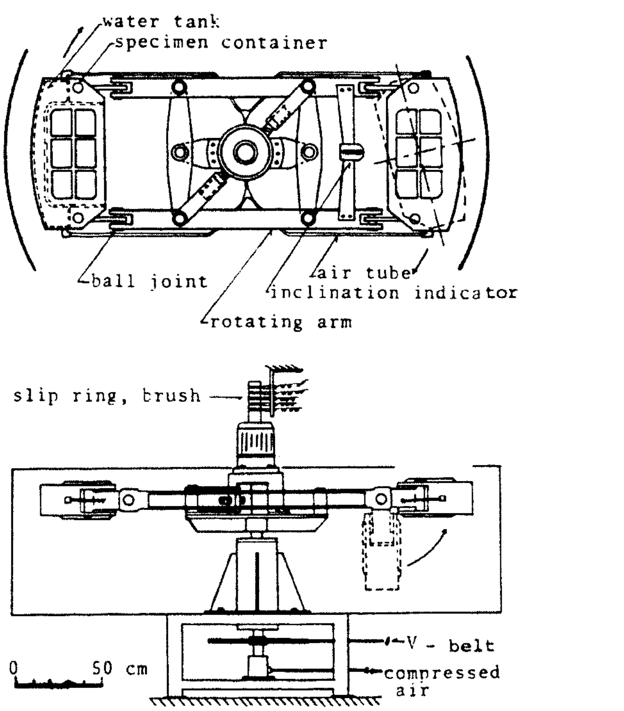

Fig.1 Tilt system on Osaka centrifuge, Mikasa et al.

The model chamber was physically tilted slowly back and forth in flight to create a cyclic lateral acceleration field, uniform throughout the model. Clearly this is not a particularly versatile technique but in problems such as dry slope stability this approach was clearly an ingenious development.

One of the simplest forms of dynamic actuator is a resonant system in which a model and a reaction mass are connected through a spring system which is primed and then released in flight. This concept was used by Morris at Cambridge University, Morris (1979), with a strongbox and ring beam reaction mass connected by steel plate springs, Fig.2. Immediately prior to the flight the reaction beam and the strongbox were jacked apart and a 'snap-through' catch was inserted to hold them in place. In flight a hydraulic ram ejected the catch and the system vibrated with a broadly sinusoidal motion, decaying with time.

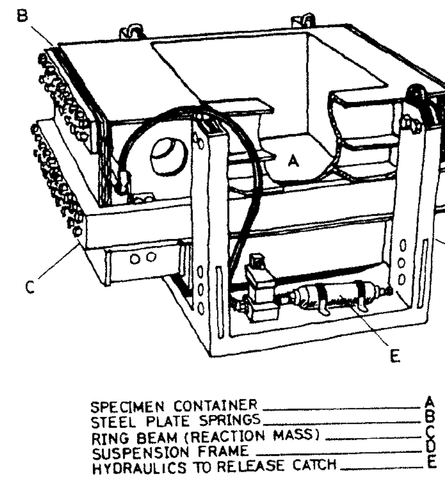

Fig.2 The Morris Box.

The same system was used by Ortiz et al. (1983) on the Genisco centrifuge at Caltech to generate one or two cycles of shaking on models of retaining walls. In this case the centrifuge itself was used as the reaction mass, Fig. 3.

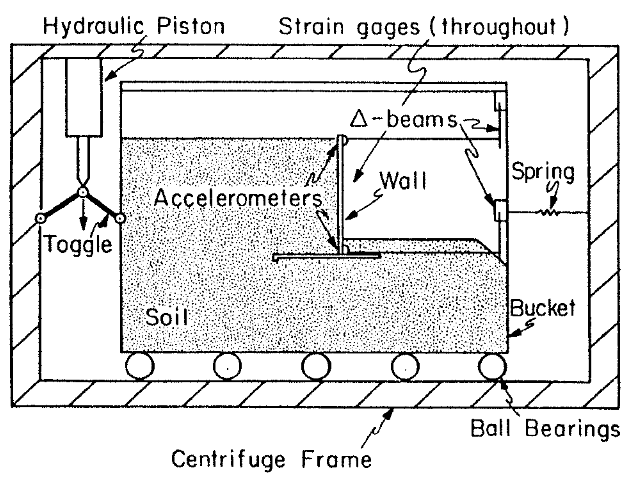

Fig.3 Snap through catch, Ortiz et al.

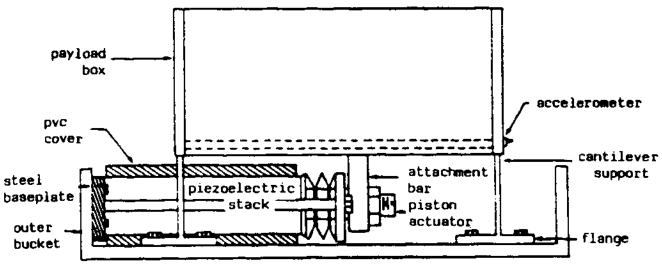

Fig.4 Piezoelectric shaker, Arulanandan et al.

At UC Davis a piezoelectric shaker was developed, Arulanandan et al. (1982) capable of sinusoidal excitation at around 500 Hz on a soil sample up to 23 Kg at 100g. Fig. 4 shows the piezoelectric stack and the model containment. The shaker exploited resonances in the drive system which greatly reduced the required output power. The system could be tuned to generate a shaking frequency as low as 100 Hz.

Morris's work had investigated the response of rocking towers on a dry sand bed. In France centrifuge models were also being used around this time to study earthquake induced soil-structure interaction for the nuclear industry. A system of generating dynamic shaking using a sequence of small explosive charges in a blast chamber at one end of a model strongbox was developed at the Centre d'Etudes Scientifiques et Techniques de l'Aquitaine for the 1 tonne, 100g CESTA centrifuge, Zelikson et al. (1981). The air blast from the chamber, Fig. 5, is directed via baffles onto a rubber membrane supporting the soil. The coffin-like shape of the model strongbox was designed to provide a central area of 0.25 m diameter within which boundary effects were small. In its spectral content, the shaking that can be generated by the explosive system compares favourably with spectra from historic earthquakes. It remains one of the few systems to deliberately generate vertical as well as horizontal accelerations. However, in many laboratories the regular use of explosives to generate shaking on a model would be impractical regardless of the success or failure of the approach.

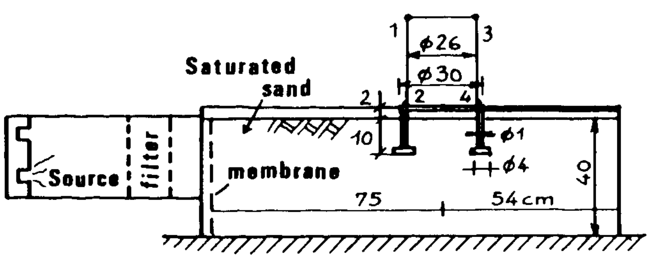

Fig.5 Explosive shaker, Zelikson et al.

In Cambridge the Bumpy Road earthquake actuator was commissioned in 1980. Schofield (1981) discussed in some detail its design and operation; the system was fully reported in the thesis of Kutter (1982). For the purposes of this discussion the Bumpy Road system may be classifed as a resonant mechanical shaker. Motion of the strongbox is generated through a lever mechanism by a wheel following the profile of a track fitted to the wall of the centrifuge pit, Fig. 6.

Consider a prototype sinusoidal oscillation at 1 Hz. At 100g the time scale for inertial events would give a testing frequency of 100 Hz. To provide a lateral acceleration of ±20% at 100g would require a base input of ±20g, which at 100 Hz is a physical movement of only $a = 20 \times 10/\omega^2 = 200/3.95 \times 10^5 = \pm 0.5$ mm . By comparison the half amplitude of the track is 2.68 mm providing a lever action of about 5.

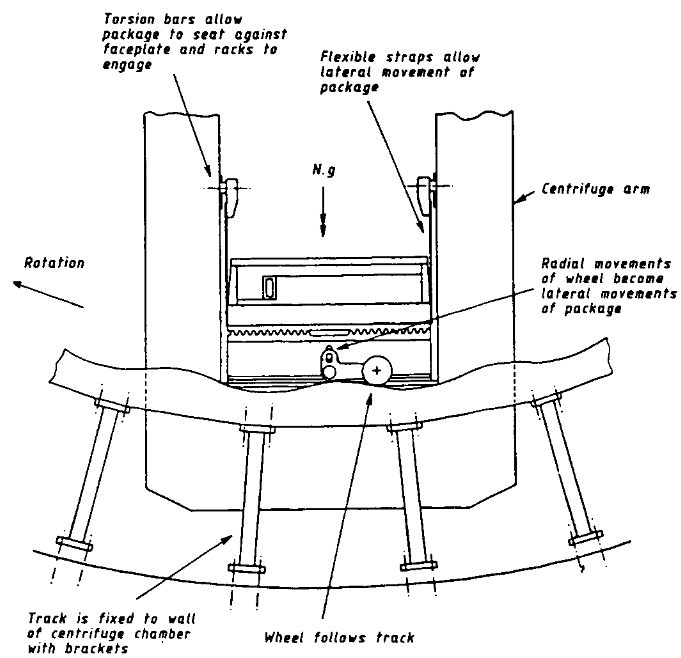

Fig.6 The Bumpy Road system.

The maximum shaken payload of the Bumpy Road system is 287 kg, of which around 160 kg is the original strongbox, leaving 127 kg for the model itself. The shaken mass reacts against the centrifuge arm and in practice the system has proved to be highly non-linear in its response due to resonant amplification. Although difficult to predict, over the years this has clearly been one of the strengths of the system in retrospect.

Dynamic excitation is primarily about energy, both energy storage and peak available power. The Bumpy Road derives its energy from the kinetic energy of the boom itself; with a nominal radius of R = 4 m the Cambridge centrifuge needs to rotate at 151 rpm to generate 100g. Therefore the stored kinetic energy in the 15 tonne beam is approximately KE = $R^3m\omega^2/3$ = 10 MJ , where m is the mass per metre. The instantaneous peak power needed to shake the model and its container is approximately 22 KW (20% acceleration at 134 Hz on 300 kg at 100g) which is clearly insignificant in relation to the beam but large in comparison to the power available from most of the other existing shaking systems.

There are consequences for the machine itself; the impact of the wheel on the track also causes a substantial shock load to be passed into the beam and thence to the main bearings. Each episode of shaking maginally reduces the life of the facility.

Clearly the Bumpy Road was a unique solution to the needs of a particular facility, and its success has been a testament to its designers, Mr P W Turner, Dr R G James and his then research student Dr B L Kutter, and to its operators, Mr C H Collison in particular. In less than ten years the Bumpy Road has formed the major part of eight PhD Theses alone of the Cambridge Soils Group, with two more currently in preparation.

During the same period the development of shaking systems for centrifuges in the US and Japan has concentrated on the use of servo-controlled electro-hydraulic systems to provide nominally programmable base shaking, at least at low frequencies.

The first of these was developed for the 1 m radius, 4.5 g-tonne, Genisco centrifuge at Caltech in 1981/82. Subsequently electro-hydraulic shakers were assembled in the US at UC Davis for operation on their Schaevitz centrifuge and at the University of Colorado at Boulder for operation on their Genisco centrifuge.

The principles and basic design of the electro-hydraulic shaker were well described by Ketcham et al. (1988) from which Fig. 7 has been reproduced. Under a flat shaking (or slip) table a large actuator is mounted. The actuator drives the table, which is mounted on linear bearings, "horizontally" in flight, reacting against the mass of the swing basket. The direction of motion induced by the actuator is parallel to the axis of the centrifuge itself, reducing errors which may be introduced by Coriolis effects (discussed below) or, on small radius centrifuges, by the curvature of the g-field. On this facility a long model mounted on the table is subject to a parallel g-field throughout its length.

Ketcham et al. noted the differences between displacement input and output signals found during early experiments at 1g and pointed to difficulties in controlling the level of noise in the system above about 100 Hz.

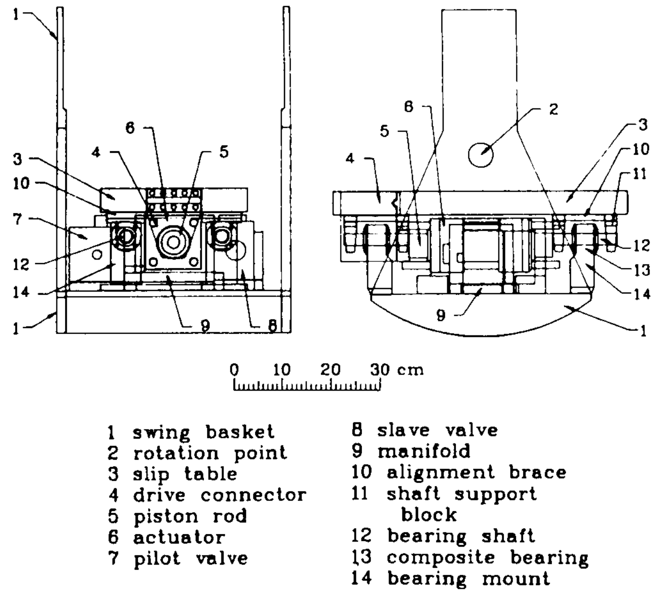

Fig.7 Electrohydraulic shaker, Ketcham et al.

In Japan a highly active group of centrifuges has developed, many of which have a dynamic capability. Electro-hydraulic shakers have been developed for centrifuges including Tokyo Institute of Technology, shown in Fig. 8 from Kimura et al. (1988a); the Port and Harbour Research Institute, Inatomi et al. (1988); Toyo Construction Technical Research Institute, Akamoto and Miyake (1989); the Public Works Research Institute, Koga et al. (1988), and the Disaster Prevention Research Institute at Kyoto University, Kita (1989).

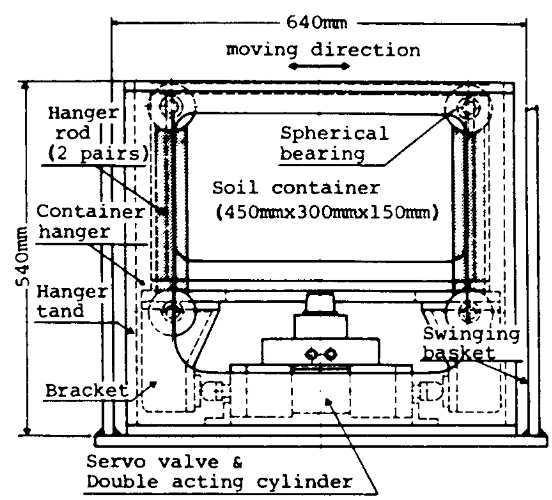

Fig.8 Electrohydraulic shaker, Kimura et al (1988a).

An electromagnetic shaker has been developed at Chuo University, Fujii (1991) which has generated sinusoidal shaking above 300 Hz on a 120 kg payload at 50g. The principal of the shaker is shown in Fig. 9; the AC coil acting as the exciter coil and the DC as the moving coil. As the required displacement is small the air gap between the coils can also be small, which improves its efficiency. As there is no mechanical contact between exciter and moving coil the time histories of shaking are noticeably free of high frequency noise, Fig. 10, which has caused problems for mechanical shakers such as the Bumpy Road (or oscillating cam systems described below). The system developed at Chuo is capable of exerting 20 kN of lateral force (or a maximum acceleration of 10g) on a payload of 200kg at 50g.

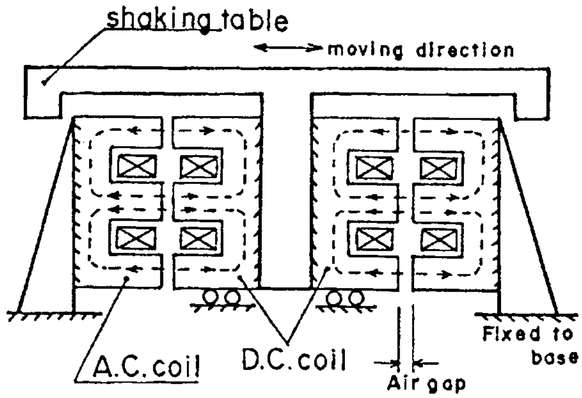

Fig.9 Electromagnetic shaker, Fuji.

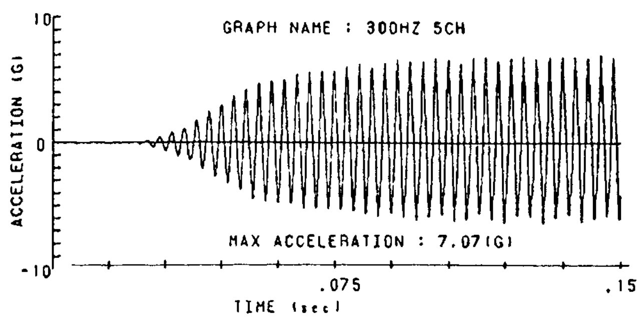

Fig.10 Data from proof tests, Fuji.

Two further mechanical shaking systems complete the picture. The first of these, on which several shakers have been based, is the rotating eccentric cam mechanism, for example at Tokyo Institute of Technology, Kimura et al. (1988b) or in the form of the oscillating link shaker at Cambridge University. Figs. 11 and 12 illustrate the mechanism and reproduce data from the TIT shaker. Clearly the eccentric cam is highly restricted in the motion that it can generate but in this form it remains a very economical solution to providing a basic shaker. Indeed the rotating cam is directly analagous to the oscillating wheel of the Bumpy Road system and it is not surprising that the time histories of shaking look very similar, Fig. 12.

For large packages it may be difficult to provide sufficient energy directly through a motor set and development work at Cambridge University is now investigating the use of counter-rotating flywheels to store energy prior to the shake.

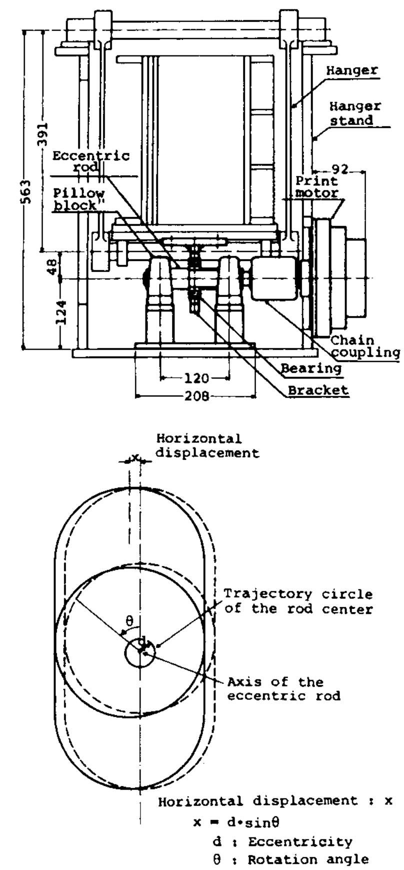

Fig.11 Eccentric cam shaker, Kimura (1988b).

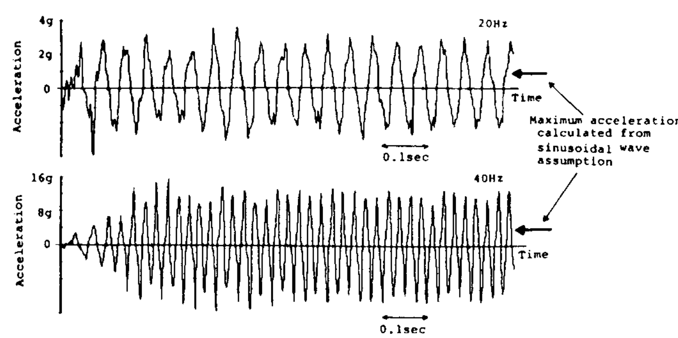

Fig.12 Proof tests of the cam shaker, Kimura (1988b).

As a final example of yet another concept of shaking a very simple hammer-plate exciter was developed at Princeton University for use on their 40in centrifuge. Fig. 13 shows the principle underlying the system which was described in detail by Coe et al. (1985).

Elastic waves propagate into the soil due to the vibrations of the plate. The approach is therefore very similar to the moving boundary used by Zelikson, except that in Zelikson's model the boundary was vertical and excited by explosives. In the Princeton box the plate boundary is horizontal and the excitation is a single hammer blow.

Both research groups have noted difficulties with standing waves within the chamber producing highly non-uniform soil motions. At Princeton the small size of the bucket precluded the option adopted at CESTA of simply ignoring all but a central area. Instead considerable effort was devoted to studying boundary conditions and to limiting wave reflections within the model. The industrial filler material duxseal was found to be effective in reducing wave reflections from the walls of the model chamber.

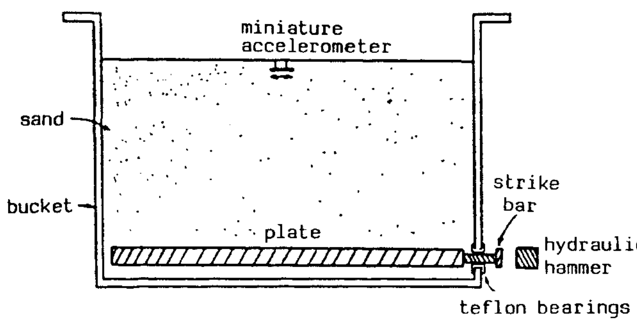

Fig.13 Hammer plate shaker, Coe et al.

The technique of using duxseal on vertical model boundaries has been generally adopted with two exceptions. The nature of the material is such that mechanical tests cannot be carried out on it in any practical manner. This has meant that it is particularly difficult to model numerically and in tests in which validation of a computer code has been a major objective the use of duxseal has proved undesirable. The alternative approach has been to ensure that the areas of interest within a model are sufficiently far from boundaries to limit boundary effects. In practice, this needs to be confirmed in each model test by using sufficient instrumentation 'in the free field' to demonstrate consistency.

The major difficulty experienced by each of these systems is to provide sufficient power to shake a large payload at high gravities. As shaking is required in the future on larger centrifuge platforms and at higher gravities, perhaps routinely at 200g rather than 50g, the applied frequencies of shaking will have to be proportionately increased to satisfy the scaling relations. The power necessary to shake a model also increases proportionately to the shaken mass and thus the demands on shakers for large centrifuges will be severe, Schofield and Steedman (1988).

The high expense involved in developing a versatile system such as the electro-hydraulic shaker for high g - large payload operation has been illustrated recently by a series of proposals to the National Science Foundation for funds to support new shaker development work at several US universities. The mechanical engineering expertise required is highly specialised and, as with many other facets of centrifuge modelling work, experimental development work is often the only satisfactory approach. These large second generation electro-hydraulic shakers are now under development. There will need to be cooperation between the different centrifuge groups within the US to arrive at the most effective system.

MODEL INTERPRETATION

A key component of the recent work published on dynamic centrifuge modelling has been the use of digital signal processing to manipulate and extract more information about the models from the data captured by the instrumentation.

This feature of model testing, perhaps above all others, is responsible for the explosion in interest and research activity over the past decade. Ten years ago signals were stored on analogue magnetic tape. Researchers were restricted to visual observations and to examination of the time histories.

Consider the evidence of liquefaction in a stacked ring apparatus at 40g reported by Heidari and James (1982), Fig. 14. These records were plotted on an analogue x-y plotter and aligned by hand. Note the different scales on the right-hand side. Nevertheless the observation of a solidification front, Scott (1986), propagating upwards towards the surface is quite clear.

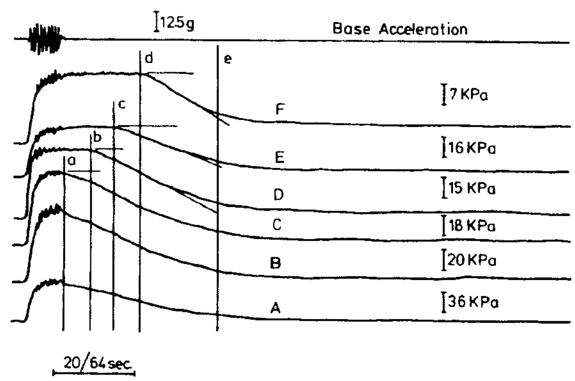

Fig.14 Data of liquefaction in stacked ring, Heidari and James.

The initiative taken towards the study of boundary conditions at Princeton University showed how spectra could be used to extract further information. Fig. 15, reproduced from Coe et al. (1985), shows clearly the beneficial effect in reducing high frequency modes of standing waves within the model as the thickness of duxseal on the vertical model walls is increased.

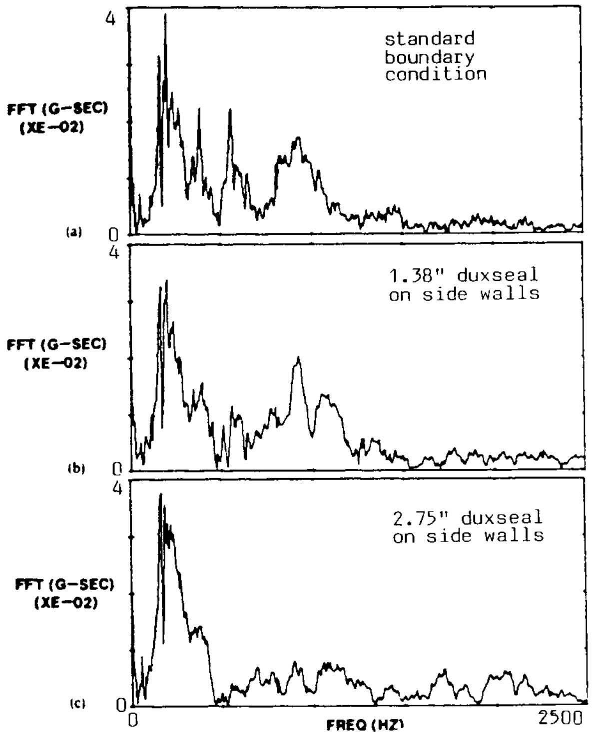

Fig.15 Effect of increasing duxseal on chamber wall, Coe et al.

Duxseal has been only one of many solutions to the rigid boundary problem. The stacked ring apparatus, referred to above, was an initial attempt to create a shear column. However, there is no evidence that the teflon coated rings actually slid relative to each other and it is more likely that the cylindrical containment structure acted in bending.

Current developments of the stacked ring concept are to investigate the use of rubber gaskets between aluminium alloy rings, with a membrane on the inner face capable of reacting the complementary shear stresses on the vertical plane. The stiffness of the gaskets may be increased with depth to mirror the increasing shear modulus in the soil, creating a 'neutral

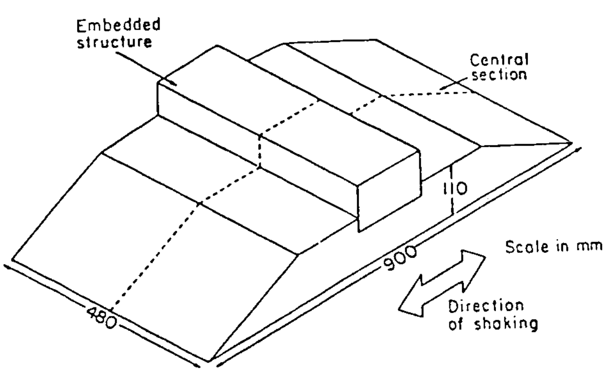

Fig.16 'Embankment' model for code validation, Finn.

shear stack'. This approach has been adopted by Cambridge and Bristol Universities under a current SERC funded research contract.

An option which has been used with some success in a long model chamber such as the Bumpy Road Box is to avoid the end wall condition altogether by using an inclined slope as shown in Fig. 16, Finn (1986). These tests were specifically designed to generate data for code validation purposes, and therefore the geometry of the model was chosen to accomodate the finite element representation, rather than to match a specific prototype.

The embankment model was used to great effect by Lee and Schofield (1988) to demonstrate the degradation of stiffness within a soil structure with excess pore pressure generation. By windowing the data and examining spectra from different accelerometers it is clear that the response of the embankment is changing during the earthquake; in this example in Fig. 17 the high amplification of motion initially exhibited at between 500-600 Hz is dramatically reduced during the latter part of the shaking to around 255 Hz.

A second key feature in interpreting the response of a soil structure is to study the phase relationships between different transducers. Lee and Schofield noted phase changes of almost 150° between the surcharge on an embankment and soil beneath it as excess pore pressures reached the level of the overburden stress.

Model tests on anchored quay wall structures such as Fig. 18 also demonstrated degradation of natural frequency in advance of liquefaction, Steedman and Zeng (1990). The response of the structure and the amplitude of bending moments in the wall were seen to depend on the relationship between the natural frequency of the soil-structure system and the frequency of the input base shaking. The onset of liquefaction in the backfill was of only secondary importance in the ensuing failure.

Such detailed observations of the evolving dynamic behaviour of a soil-structure system were possible because of the broadly harmonic input. Both examples were of saturated soil, prone to further softening by excess pore pressure generation. In the field, earthquakes on soft soil sites repeatedly show a predictable, strongly periodic motion; consider the time histories recorded in Oakland during the Loma Prieta event, for example.

Centrifuge models of the behaviour of geotechnical structures under earthquake loading have shown clearly that the degradation of stiffness prior to liquefaction is likely to dominate the response of the soil profile and consequently the response of structures founded on or within it. Such events, taking a soil profile or soil-structure system towards resonance, inevitably involves large strain cycling which in itself will lead to further softening.

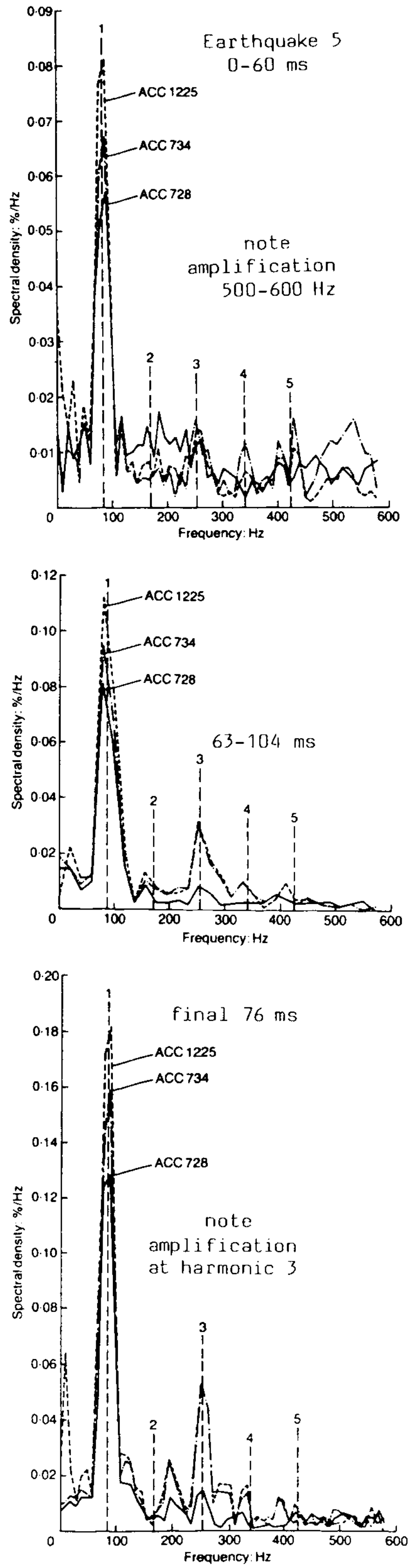

Fig.17 Fourier spectra of windowed data, Lee and Schofield.

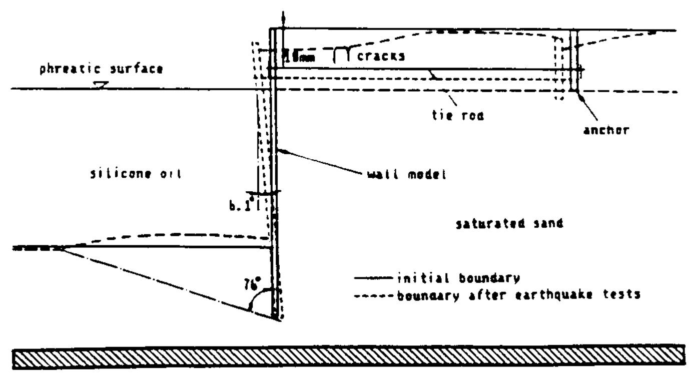

Fig.18 Failure of model anchored quay wall, Steedman and Zeng.

Interest in using the centrifuge to build a database of liquefaction induced permanent ground movements for the validation of analyses led to the three year VELACS project (Verification of Liquefaction Analysis by Centrifuge Studies), sponsored by the NSF and initiated in 1989 under the direction of Professor K Arulanandan of UC Davis. The project involves collaboration between six universities and will lead to a valuable public resource of data on soil behaviour.

BLAST MODELLING

The level of activity in the field of centrifuge modelling of blast loading has been considerably smaller than for earthquakes. However significant advances have been made from the early work reported by Schofield (1981).

As all lengths in a model are reduced in comparison to the prototype by the linear scale n then volumes, masses and therefore energy are reduced by n^3. A 1 tonne charge in the field will therefore be equivalent to a 1 gm charge at 100g.

At Boeing Aerospace in Seattle impact and explosion cratering has been modelled on centrifuges under the direction of Dr R M Schmidt. In their recent paper Schmidt and Housen (1987) describe the use of the Boeing 600g, 66 g-tonne centrifuge to study cratering events in dry and saturated sands and water. High speed photography using a Fastax II rotating prism camera allows framing rates of up to 13,000 pictures per second whilst in flight at over 500g.

The models are constructed in circular tubs to form a 'half-space' model or in a circular tub with a transparent window (a 'quarter-space' model) which can give a view of the crater formation in real time.

Impact cratering has been achieved using hypervelocity projectile guns mounted on the arm capable of projectile velocities of up to 7 Km/s.

Field data of cratering events which is in the public domain is relatively scarce and largely unsatisfactory because of the variability of ground conditions and soil profiles in the field.

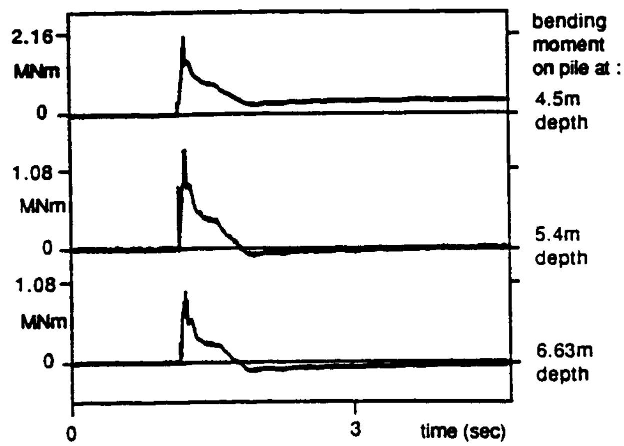

Fig.21 Bending moment at different depths of pile (prototype units).

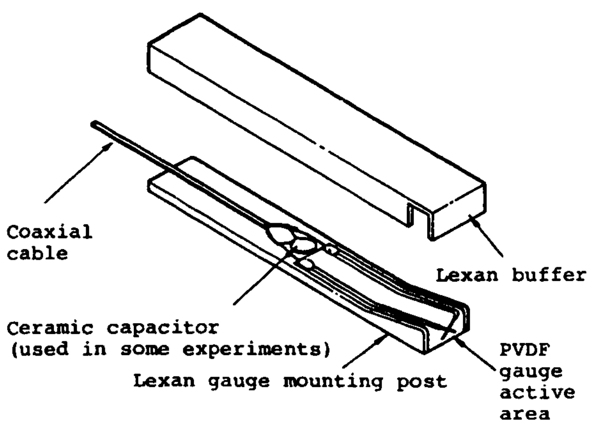

Fig.22 PVDF total stress gauge, Gaffney et al.

The length of the gauge is necessary to provide restraint for the device during the passage of the blast wave. Fig. 23 shows typical results from a blast, with a shock pulse being followed by a longer period of applied pressure which reflects the period of soil flow past the gauges. In Fig. 23(a) and (b) the duration of the pulse is around 1 ms. Both of these gauges were laid horizontally in the soil bed and the duration of the pulse is limited by the time taken by the wave to travel along the gauge. Once the wavefront has totally enveloped the gauge the gauge will move with the soil and the frontal stress will rapidly fall. Fig. 23(c) however, shows data from a gauge mounted vertically near the edge of the crater with the active element buried in the soil. This gauge was bolted to a rigid frame and the longer duration of the pressure pulse (around 4 ms) is more likely to reflect the actual duration of soil flow.

The gauges were calibrated in shock tube tests by Ktech Corp. and were generally positioned in the plastic zone beneath the true crater surface. Nevertheless peak pressures of up to 25 MPa were measured at shot depth as far as 158 mm from the charge (9.5 m in prototype dimensions).

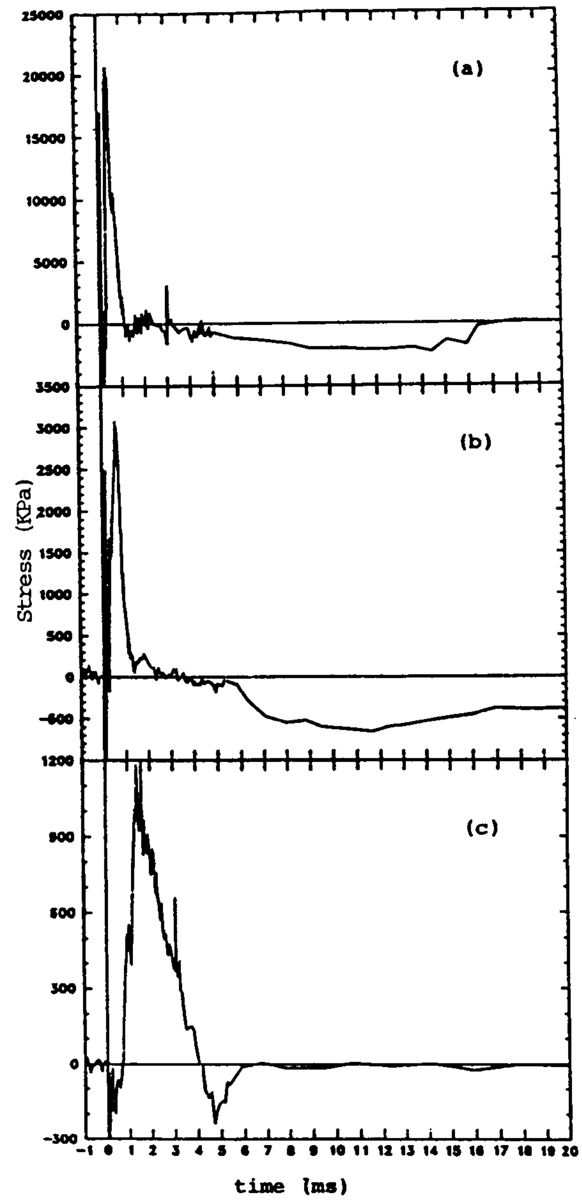

Fig.23 Data of total stresses, Gaffney et al.

Such a large initial total stress wave causes significant difficulties in the use of pore pressure transducers to measure transient pore pressures during the period of soil deformation following the blast. Conventional Druck miniature pore pressure transducers at a similar range consistently showed a pressure pulse of the order of 300-500 KPa lasting approximately 1 ms. The transducers were aligned in some cases directly facing the source and in others at right angles to the ray path but no strong influence on the magnitude or character of the pulse was observed.

However it is well established that two broad regimes dictate the efficiency of a crater, which is defined as the ratio of ejected mass to charge mass or, following Schmidt and Holsapple (1980), $\pi_V = V\rho/M$ where V is the crater volume, ρ the soil density and M the charge mass. At low yields the strength of the material dominates the size of the crater and this is indicated by a cratering efficiency that is independent of the scaled yield, as indicated in Fig. 19. Scaled yield is defined by the non-dimensional group $\pi_2 = G(M/\delta)^{1/3}/Q$ where M is the charge mass, δ the initial density of explosive, Q the specific energy and G = ng is the acceleration field.

However, at larger yields gravity effects dominate the behaviour and the efficiency is reduced. Centrifuge experiments enable this transition to be mapped in some detail with repetitive model tests in identical soils.

The data shown in Fig. 19 show the results of model tests by different researchers. The data of Schmidt and Housen were obtained from explosive tests in saturated sand in a quarter space model. The charge was usually spherical and just covered by soil. The data of Serrano et al. (1988) represent experiments in dry sand using a cylindrical charge laid horizontally and half-buried. The data from Steedman (1989), were taken from measurements of the observed craters during model tests investigating blast loading on piles. The experiments used saturated sand with a spherical charge buried at optimum depth. The final data point on the figure was taken from James (1978) and represented a surface burst in saturated loose sand at 100g.

It is clear that variations in the level of the water table, the density of the sand, the shape and depth of the charge will all have had a significant effect on the final crater volume. Superimposed on the data are Schmidt and Housens' proposed scaling laws for gravity dominated impact craters in "dry and wet sand", based on their own data.

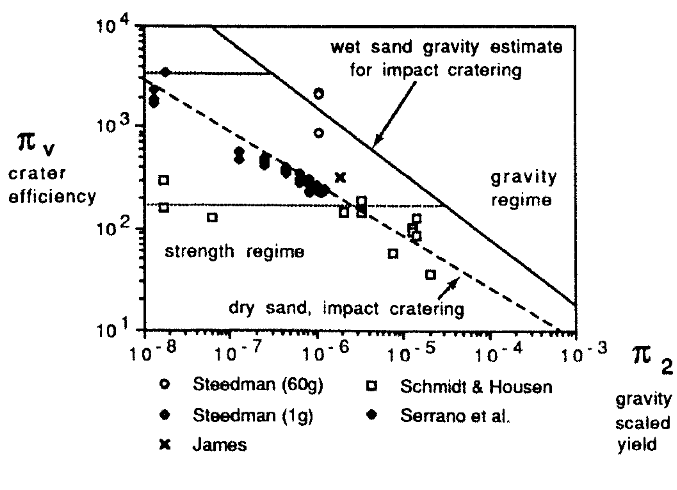

Fig.19 Crater data for wet and dry sand.

Although the dry model tests of Serrano et al. show good agreement with the empirical relations clearly there is much greater variation in crater volume amongst the saturated models. One of the most likely explanations for this is the phenomenon of blast induced liquefaction linked to differences in the depth of the water table, a parameter which appears to be as significant as the depth of the charge but which is not yet clearly understood.

Steedman et al. (1989) reported tests at 60g of the response of piled structures to blast loading from an adjacent buried charge. In Fig. 20 a pair of piles is shown, fixed at the ground surface, in a saturated sand bed. A 2 gm charge was detonated at a depth of 74 mm with various water table depths. Strain gauges on the piles recorded the transient bending moments as the soil flow moved past the piles, Fig. 21.

The pressure wave caused by the blast was captured by total stress gauges such as those shown in Fig. 22. These were relatively large units with active elements comprising biaxially stretched, hysteretically poled polyvinylidene flouride (PVDF) film 26 µm thick. The film is manufactured by a French company Metravib RDS and the gauges were assembled by Ktech Corp. of Albuquerque.

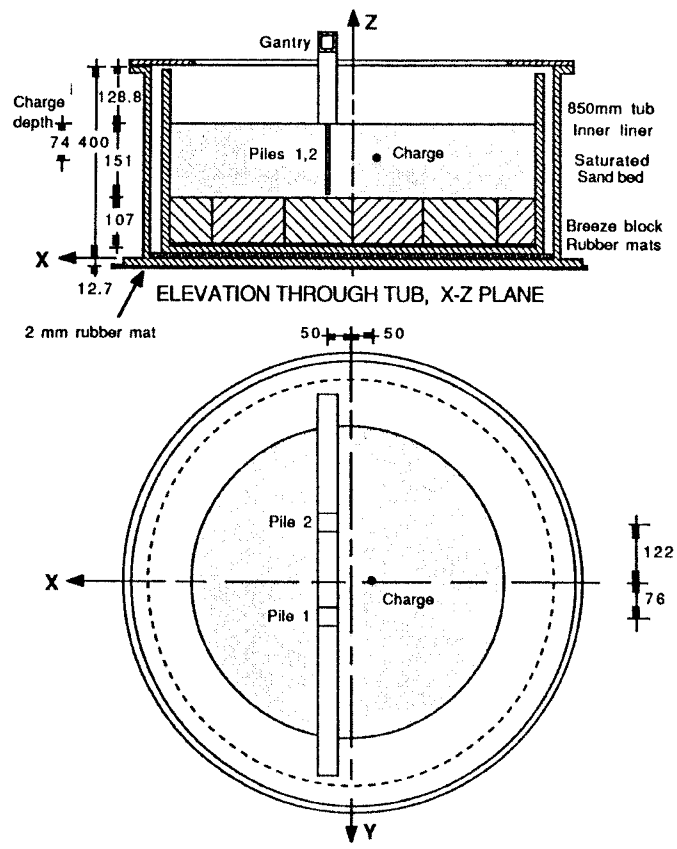

Fig.20 60g model piles subject to blast load.

Clearly in the post shock wave environment excess pore pressures such as those associated with liquefaction will be limited to approximately the overburden pressure which is likely to be only tens of Kilopascals rather than hundreds or thousands. However a transducer which is sufficiently robust to withstand a Megapascal shock will generally be rather insensitive and this will affect the quality of data captured in this phase.

Lee (1990) has studied the frequency response of diaphragm pore pressure transducers of the type used for dynamic centrifuge model testing. It is common in centrifuge modelling of the earthquake behaviour of saturated sands to use a pore fluid such as silicon oil, or a glycerine/water mixture, which has a higher viscosity but similar density to water. Although in many geotechnical problems the undrained and drained phases can be separated for the purposes of analysis, earthquake loading on saturated sands is a problem of coupled consolidation and identical time scales for inertial and diffusion events are desirable.

If the grain size used in the model is identical to the prototype then the pore fluid viscosity requires to be increased by the linear scale n to reduce the time scale for diffusion events from n^2 to n, in line with the time scale for inertial events.

Lee analyses the dynamic response of a typical Druck PDCR 81 pore pressure transducer with frequency, varying the viscosity of the pore fluid. For a completely saturated transducer it was found that the ratio of measured to actual pressure amplitude was dependant on the viscosity of the pore fluid. Fig. 24 plots the results of Lee's calculation, extended to higher frequencies and concentrating on the most commonly used range of pore fluid viscosities. The response for water as a pore fluid is flat to above 10 KHz, indicating that the viscous damping effect of the porous cap is negligible.

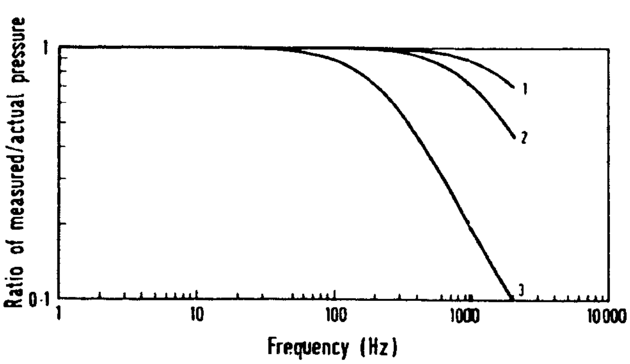

Fig.24 **Frequency response of saturated PPT with viscosities 50 cS (1), 100 cS (2), 500 cS (3), Lee.**

In blast modelling of saturated sands researchers to date have generally used water as a pore fluid. In Fig. 20 two pore pressure transducers are shown at shot depth and on the same ray path, a 35 bar device at a range of 160 mm (PPT 3743) and a 3 bar unit (PPT 2567) at a range of 197 mm from the charge. Fig. 25 shows the arrival of a pore pressure pulse at the two transducers together with the time history of bending strain on one of the piles, at a depth of 40 mm and range of 125 mm.

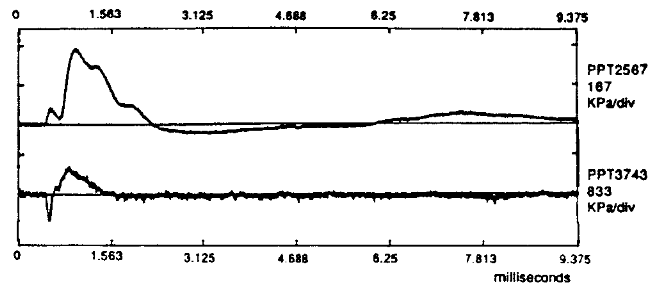

Fig.25 **Pore pressure wave during the initial shock wave.**

Rise times for the pore pressure pulses are short, of the order of 0.24 ms, equivalent to a frequency of around 4 KHz. Using oil as a pore fluid in this case would therefore introduce a significant error in the measurement of the peak transient pore pressure.

There is a brief but distinct delay in the arrival time of the pressure wave between the two pore pressure transucers. From the data this was measured as approximately 99 µs, giving an average wave propagation speed of 1400 m/s, close to the speed of compression waves in water (1500 m/s). The ratio of the ranges of the outer to the inner PPT is 297/160 ≈ 1.9 ; the duration of the pulse has increased roughly in the same proportion, by about 2, whilst the amplitude has reduced with range. However another source of error is likely to be caused by the small size of the transducer body which may have affected the measurement of the peak amplitude of the passing wave, particularly as in this case the transducers were aligned differently, the inner one being perpendicular to the ray path and the outer one parallel.

The more sensitive 3 bar PPT clearly shows a dilative wave following the compressive front. This period is indicative of soil shearing and corresponds broadly with the long duration of transient lateral pressure observed at the pile (BMT1040) on the othe side of the crater.

Although difficult to observe in the high g tests because of the background noise there was strong evidence from an identical model carried out at 1g of a third phase, a consolidation phase, which followed the shearing. Fig. 26 shows the dilative and consolidation phases for two pore pressure transducers at the same range (≈ 160 mm) but different depths, PPT2777 at 32 mm depth and PPT2810 at 106 mm. The initial positive pulse has been removed for clarity. At depth pore suctions of 1 atmosphere (100 KPa) stop abruptly to be followed by a period of pore pressure redistribution. As the shearing phase ends with a net pore suction fluid must be drawn downwards from above (in contrast to the conventional image of liquefaction in which pore pressures are expelled upwards). The upper transducer shows a longer period of shearing before this too ceases and redistribution takes place.

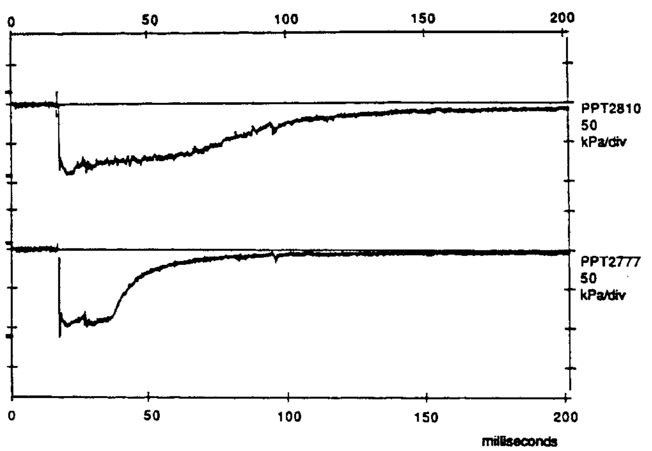

Fig.26 Dissipation following shearing, seen at 1g.

The nature of this dissipation event can be confirmed by simple calculation. The time for 95% consolidation in a 1 dimensional bed is approximately

$$t = d^2/C_o \qquad (1)$$

where d is the depth and $C_o = E'_o k/\gamma_w$, Terzaghi's coefficient of consolidation, with E'_o the constrained effective elastic modulus, k the permeability and γ_w the unit weight of water. Adopting the widely used relationship for small strain shear modulus G_{max} as a function of void ratio e and mean effective confining pressure σ'_m from Hardin and Drnevich (1972)

$$G_{max} = \frac{3230(2.973-e)^2}{(1+e)}\sqrt{\sigma'_m} \qquad (2)$$

gives a value of approximately G_{max} = 6.3 MPa at 74 mm depth. Using a Poisson's ratio of υ' = 0.25 gives a value for $E'_o = 3G_{max}$ = 18.9 MPa. The permeability of the aggregate can be estimated from the particle size distribution using empirical relations such as $k \approx 10000(d_{10}^2)$ m/s for Leighton Buzzard sand. Leighton Buzzard 100/200 sand has a d_{10} = 0.09 mm (ie. 10% finer than 0.09 mm) and hence $k \approx 8.1 \times 10^{-5}$ m/s.

The time for 95% consolidation is therefore around 37 mS, which compares favourably with the decay shown in Fig. 26.

At 60g, however, the stiffness of the soil will be roughly $\sqrt{n}$ (≈ 8) times greater than at 1g. The permeability group k/γ_w is constant (both k and γ_w increasing by n) and therefore the consolidation at 60g in a model of identical geometry should be approximately 8 times faster, giving a time of around 4.6 ms at 60g.

Following the theory of parabolic isochrones drainage at the same location will commence at a time of about 1/12 of this figure, or 0.4 ms. It is clear from the durations of the events shown in Fig. 25 that these must be partially drained.

The use of water as a pore fluid therefore has advantages and disadvantages. In earthquake modelling there is clear evidence of the coupled nature of events in saturated sand models and therefore of the necessity to correct time scales either by using silicon oil, glycerine or by changing the grain size distribution. In blast modelling the analysis above demonstrates that using water as a pore fluid for even these very rapid events may lead to partial drainage during the shearing phase at least.

This section started from a consideration of apparent crater volume. The volume of the craters formed in the saturated sand models was dominated by the depth of the water table. In the 60g test described above the apparent crater was extremely shallow hiding a deep liquefied bowl, Fig. 27. Sand markers clearly identified the limits of the 'bowl' and traced the permanent strains in the surrounding soil. The piles were placed at ranges of 125 mm and 160 mm to be near the lip of the crater and there was clear evidence of scour around the inner pile, like a rock within a stream.

The model piles subject to the blast loading were designed to have the correct scaled bending stiffness compared to the prototype piles but were fabricated from aluminium alloy tube which gave them a higher scaled plastic moment capacity. This had the advantage that bending strains could be accurately tracked along the length of the pile, and some interpretation made of the bending moment at each location. The piles were held rigidly at the ground surface and failed with the formation of plastic hinges just below the ground surface and around shot depth, although clearly their failure load was considerably in excess of the prototype piles.

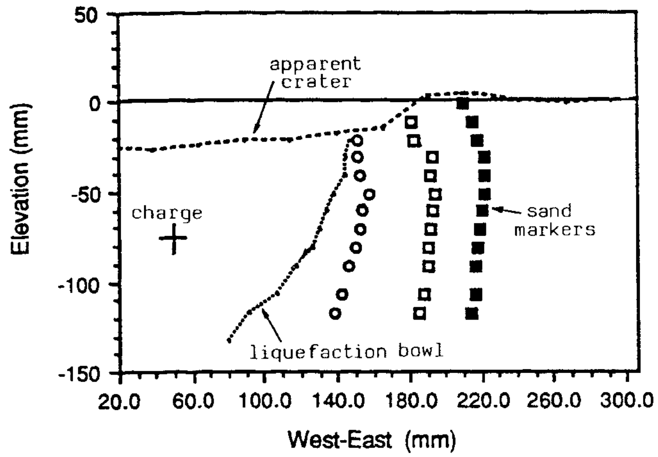

Fig.27 Cross section through crater in saturated sand at 60g.

Other materials have also been used in dynamic modelling. Bolton and Steedman (1982) describe the use of microconcrete to construct retaining walls for earthquake model tests in the 'Morris Box'. Townsend et al. (1988) investigated the protection to a buried box structure provided by a microconcrete burster slab in centrifuge tests at 60 and 82g. Kutter et al. (1988) reported centrifuge tests of blast loading on buried tunnels fabricated from brass sheeting or using aluminium beverage cans. All of these tests were carried out in dry sand models.

Centrifuge modelling of blast effects has advanced to a new level, in which model structures are instrumented and soil-structure blast interaction is observed. The precise nature of the transfer of energy via a shock wave passing through the ground and into a structure can now be explored.

CORIOLIS EFFECTS

As the range of centrifuge applications widen and research groups in new fields turn to the centrifuge for realistic physical data potential limitations such as Coriolis effects will require to be comprehensively addressed. Both earthquake and blast experiments potentially suffer from Coriolis effects and this has been commented on in the literature, for example Schofield (1981). There are a number of examples of dynamic models, including models involving penetration such as Fragaszy et al. (1988), which create velocities of the right order for sufficiently long to cause a significant distortion. Current proposals for new areas of research include high velocity hydraulic models, wave loading and models of structural dynamics, all of which may also find Coriolis effects unavoidable.

Coriolis effects necessarily arise from the transformation between an inertial (or Newtonian) coordinate system and a rotational system. Particles moving within a model on a centrifuge are in equilibrium in a coordinate system accelerating relative to the earth. Viewed from the earth they do not appear to obey Newton's Second Law of Motion. To interpret the motion of a particle in an accelerating system its equation of motion becomes

$$\underline{F} - 2m(\underline{\omega}\times\underline{v}_r) - m\underline{\omega}\times(\underline{\omega}\times\underline{r}) = m\underline{a}_r \qquad (3)$$

where $\underline{r}$, $\underline{v}_r$ and $\underline{a}_r$ are the position, velocity and acceleration vectors of the particle relative to the rotating system and $\underline{\omega}$ is the (constant) angular velocity. The two additional terms are the Coriolis and centrifugal accelerations.

The Coriolis force is proportional to the cross product of $\underline{\omega}$ and $\underline{v}_r$ and is of magnitude

$$|\underline{F}_c| = 2m|\underline{\omega}||\underline{v}_r|\sin\theta \qquad (4)$$

with its direction following a right-hand rule, perpendicular to both $\underline{\omega}$ and $\underline{v}_r$. Clearly if $\underline{\omega}$ and $\underline{v}_r$ lie in the same direction or if the particle remains at rest in the rotating system the Coriolis force in zero.

Schofield (1981) calculated the ratio of the magnitude of the Coriolis acceleration to the steady centrifugal acceleration

$$\frac{A_c}{A} = \frac{2\omega v}{R\omega^2} = \frac{2v}{V} \qquad (5)$$

where R and V define the radius and tangential velocity of the model container, and used this to define a range of particle velocities which will be prone to distortion. Clearly if v becomes greater than 5% V, for example, then the error in neglecting Coriolis exceeds 10%. Particles with a high velocity will also be subject to only a small distortion as they travel fast relative to the motion of the container. Following Pokrovksy and Fyodorov (1968) then the distortion caused by Coriolis will be equal to distortions due to the radius of the model container when $v = 2V$.

Thus for a model under 60g at a radius of R = 4m Coriolis might be expected to cause significant distortion for particles travelling at velocities between 2.5 m/s and 97 m/s.

The trajectory of a particle ejected at a velocity v and at an angle ϕ from the instantaneous direction of travel of a model can be predicted quite simply. The path of the particle relative to the centrifuge model container is obtained by transforming its actual path into local coordinates. Fig. 28 defines a local x,y coordinate system; it can be seen that a particle given an initial velocity $v = \sqrt{(v_x^2+v_y^2)}$ travels along a straight path which is the vector sum of the tangential velocity of the package and the velocity v. The angle which the actual or absolute path makes to the initial

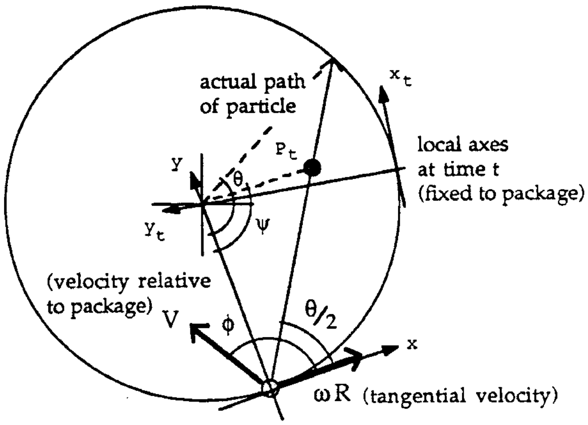

Fig.28 Coordinate system for Coriolis analysis.

x axis is $\theta/2$. As the particle travels across the circle the package moves around the circumference. The trajectory of the particle can then be plotted as it would be seen by an observer travelling on the package.

The transformation of axes from x,y to x_t,y_t is caused by a rotation $\psi = \omega t$ and translation x_T, y_T of the original system. The transformation is therefore simply given by

$$x_t = x\cos\psi + y\sin\psi - x_T\cos\psi - y_T\sin\psi \quad (6)$$
$$y_t = -x\sin\psi + y\cos\psi + x_T\sin\psi - y_T\cos\psi \quad (7)$$

From Fig. 28 it is clear that

$$x_T = R\sin\psi \quad (8)$$
$$y_T = R(1-\cos\psi) \quad (9)$$

The coordinates of the particle x,y during its flight relative to the global system are given by

$$x = V_a t\cos\theta/2 \quad (10)$$
$$y = V_a t\sin\theta/2 \quad (11)$$

where the absolute velocity

$$V_a = \sqrt{\{(\omega R+v_x)^2+v_y^2\}} \quad (12)$$

and

$$v_x = (\sin\theta-\theta)\omega R/\theta \quad (13)$$
$$v_y = (1-\cos\theta)\omega R/\theta \quad (14)$$

Figs. 29 and 30 show the different trajectories followed by a family of particles firstly with a varying angle of ejection but equal ejection velocity and secondly with a varying ejection velocity but equal ejection angle.

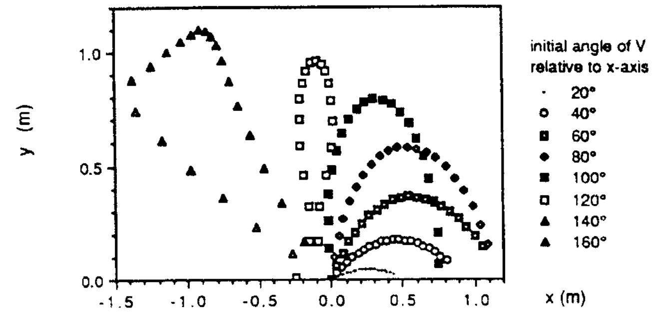

Fig.29 Trajectories with constant velocity.

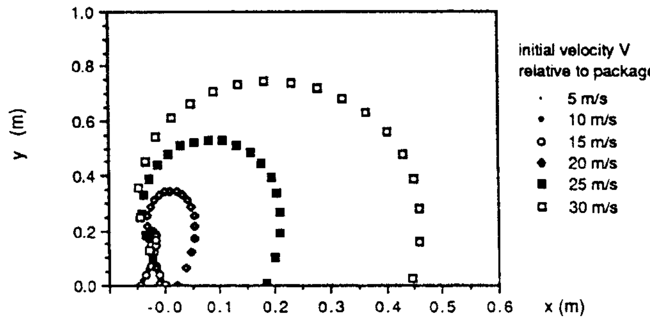

Fig.30 Trajectories with constant ejection angle.

In practice wind turbulence and the influence of the earth's vertical 1g field will affect these theoretical trajectories to some extent but it serves clearly to illustrate the phenomenon of ejecta being thrown 'forwards' in flight.

The specific example used for the calculation corresponds to the geometry of one of the model tests described above, similar to that shown in Fig. 20.

There is other evidence of the velocity of flow during the crater formation which can be compared with the trajectory predictions. The total stress measurement made in Fig. 23 showed a peak upward pressure of 1.4 MPa. Assuming that the dynamic pressure within the flowing soil is given by ρc^2, where ρ is the density and c the velocity then this would give a peak velocity at the edge of the crater of around 27 m/s.

After the blast it was clear that there had been an upward flow around the gauge. The time history of the dynamic pressure shows a roughly triangular response, with a rapid rise to a maximum and a slower decay from the peak. The overall duration of the pulse was around 4 ms. As the velocity is proportional to the square root of the pressure, the time history of ground velocity may be inferred to be parabolic, with the velocity at 1/4 of the peak pressure already up to 1/2 of its peak value, and so on. The total area under the velocity versus time graph is then approximately 4ab/3 where a is the duration of the pulse and b the peak velocity. This gives a peak displacement of 72 mm, which is of the right order based on measurements of the model.

The craters may also be assessed for 'roundness' by superimposing on a contour plot a set of circles centred on the charge location. Fig. 31 shows the results of such a plot with areas where the contour fell outside the circle shaded. It is clear that there is no strong bias in the shape of the crater. At the top of the figure a line of six piles has forced ejecta to either squeeze through between them or pile up by the rigid gantry frame at either end.

Finally, there is one further source of information on particle velocities. In cross-section the model was contained within a circular tub with an extension piece to reduce wind turbulence. Some ejecta was caught on horizontal ledges on the leading side of the container, at the joint between the tub and the extension and on top of the extension itself, Fig. 20.

Following the analysis above particles which landed on these ledges would require ejection velocities of at least 20-30 m/s. From Fig. 29 particles with a velocity of 30 m/s which were ejected between 40° and 75° to the horizontal would be caught but at 20 m/s this would reduce to only particles ejected at around 75°. There are very few combinations of velocity and angle which would lead ejecta to be caught on ledges on the trailing side of the container.

Clearly the expanding gases do not constitute a point source on the surface, as was assumed in the calculation but nevertheless there is good agreement with the pressure gauge data. It may then be concluded that in these experiments a velocity of around 20-30 m/s, measured at the edge of the crater, marked the limiting velocity of upwardly moving particles between those which became airborne and those which remained part of the soil flow.

Coriolis effects are clearly present in certain classes of model test. In the experiments reported here there was no obvious evidence of distortion to the ground flow or to the crater shape although clearly the air-borne ejecta were markedly distorted.

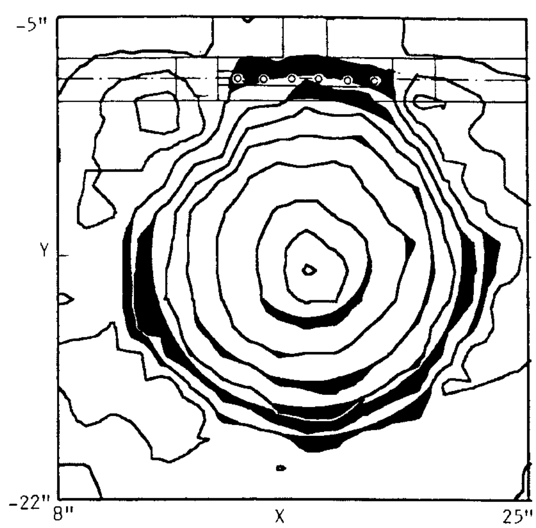

Fig.31 Contour plot of crater at 60g, showing minimal distortion.

CONCLUSIONS

There has been considerable development of dynamic geotechnical model testing on centrifuges worldwide over the past decade.

Models have become more complex and the interpretation of data more sophisticated as techniques of digital signal processing and high speed data capture have become widely available. As the costs of computing power reduce it is expected that this process will advance still further leading to a situation where data is being captured considerably faster than it can be analysed. It is expected that as centrifuge research groups gain experience of model testing analytical researchers will be required to complement the experimental workers. This will lead to pressure to archive data of dynamic tests in a detailed and accessible form.

Evidence of phenomena such as the degradation of soil due to excess pore pressures and strain softening has been observed in great detail. Degradation has been seen in centrifuge model tests to dominate the response of many classes of soil structure to earthquake loading. However this has yet to be adequately recognised in the field of design.

Blast modelling has demonstrated its potential to allow detailed examination of the formation of a crater, the transmission of energy through the ground and the interaction between soil and structure. In an area where physical evidence is critical and field tests are extremely expensive it is expected that the opportunities provided by the centrifuge will be increasingly exploited over the coming years.

In conclusion, the past decade has seen the research experience and capabilities in dynamic centrifuge modelling broaden to a point where widespread application to field problems is inevitable. This is to be welcomed as, amongst other reasons, it will provide pressure to validate the wide range of increasingly sophisticated numerical models now available. Far from diluting the available funds for research, the international development of the centrifuge as a tool for advanced modelling will build confidence and credibility with sponsors and industry alike.

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Within all fields of research there are individuals who have made a unique contribution to the subject and the author acknowledges the remarkable influence of his colleague Professor A N Schofield of Cambridge University, who has continued to encourage and cajole researchers and sponsors worldwide throughout the past decade.

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